The impact of microfinance institutions on informal credit markets

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Abbreviations and notations

\( C_i / C_M \)  
joint liability component for risk type \( i = \{r, s\} \) / any other contract \( M \)  
(without subscript it corresponds to a pooled contract)

\( ER \)  
expected repayment

\( I \) (superscript) informal moneylender

\( ICC_i \)  
set of joint liability contracts that satisfy the incentive-compatibility constraint for borrowers of type \( i \) with equality

\( ICC_{ep} \)  
set of joint liability contracts that satisfy the ex-post incentive-compatibility constraint for borrowers of type \( i \) with equality

\( il \) (subscript)  
individual liability

\( jl \) (subscript)  
joint liability

\( LLC_i \)  
set of joint liability contracts that satisfy the limited liability constraint for borrowers of type \( i \) with equality \((i \neq j)\)

\( MFI \)  
microfinance institution

\( p_i \)  
probability of success for risk type \( i = \{r, s\} \)

\( PC_i \)  
set of joint liability contracts that satisfy the participation constraint for borrowers of type \( i \) with equality

\( Q_i(a) \)  
project return for risk type \( i = \{r, s\} \)

\( \hat{Q}(a) \)  
expected project return

\( R_i / R_i \)  
individual liability component / interest rate for risk type \( i = \{r, s\} \) / any contract \( M \) (without subscript it corresponds to a pooled contract)

\( r \) (subscript)  
risky type, i.e. borrowers with a high default risk

\( s \) (subscript)  
safe type, i.e. borrowers with a low default risk

\( U_{ii}(R_i, C_i, a) \)  
payoff / utility of borrowers with risk type \( i = \{r, s\} \)

\( \Theta \)  
proportion of risky type borrowers in the population

\( \rho^l \)  
capital cost of an informal lender

\( \rho \)  
capital cost of the MFI

\( ZPC_i \)  
set of joint liability contracts that satisfy the breakeven constraint for borrowers of type \( i \) with equality

Note: The use of male pronouns in this paper is always meant to include individuals of all genders.
1. Introduction

The widespread financial exclusion of poor communities in many developing countries has been explained by a range of adverse conditions: Unavailability of collaterals, adverse selection, moral hazard and weakness of judicial systems, which renders enforcement of loan repayment extremely costly (Stiglitz and Weiss 1981; Duflo 2013). Hence, formal banks are usually replaced by informal moneylenders who often have both an informational advantage and a wider range of possibilities to enforce the repayment of loans\(^1\). However, their interest rates are usually exorbitantly high (Zeller et al. 2001; McKernan 2002).

Aiming at the reduction of poverty, the modern microcredit movement developed a system of strict repayment rules, joint liability and dynamic incentives that increased repayment rates and thereby allowed for lower interest rates (Duflo 2013). With the increasing prevalence of microfinance institutions (MFIs), it was hoped that this competition would also reduce the interest rates for informal moneylending (Berg et al. 2013; Mookherjee and Motta 2016). Few studies have examined this empirically. However, some researchers noted that such expectations were not always fulfilled (see Stiglitz and Hoff 1990; 1998). More recently, Berg et al. (2013) and Mallick (2012) showed that an increase of microfinance coverage in Bangladesh in some villages even led to a spike of the average informal interest rate. Mookherjee and Motta (2016) claim that these findings cannot be explained coherently by the literature so far\(^2\) and set up a new model to change this.

This dissent over the interaction of informal moneylenders and MFIs demonstrates the necessity to further analyze and contrast existing theories and to clarify which differences in assumptions lead to the discrepancies in their results. Moreover, comparison with

\(^1\) A widespread phenomenon, for example, is the linkage of trade and lending which allows the moneylenders to impose a form of pressure that is not available for formal banks (Hoff and Stiglitz 1990). Duflo (2013) also reports illegal methods of coercion.

\(^2\) These explanations will be reviewed in chapter 3 and include, according to Mookherjee and Motta (2016):
1. scale diseconomies for the informal lender because of decreased demand (Hoff and Stiglitz 1998; Jain 1999);
2. cream-skimming, i.e. only those with high defaults risks borrow from informal lenders (Bose 1998, Demont (2016, 2016), (Demont 2016, 2016),
3. facilitation of collusion through increased availability of credit in case of channeling formal credit through informal lenders (Floro and Ray 1997)
4. increased demand for informal loans because of strict repayment rules and moral hazard (Jain and Mansuri 2003; McIntosh and Wydick 2005; Kahn and Mookherjee 1998)
empirical findings might allow determining which models are the best fit for specific regions and socio-economic contexts.

Aiming to contribute to this endeavor, this thesis analyzes, classifies and evaluates the model provided by Mookherjee and Motta (2016) (“the model”). In particular, it will examine its validity and elucidate why its results differ from earlier explanations for increasing informal interest rates.
2. Mookherjee and Motta's "theory of interactions between MFIs and informal lenders"

2.1. Basic characteristics of the model

2.1.1 Constraints to utility maximization of the moneylender and the MFI

On the grounds of numerous assumptions (see Annex 1), Mookherjee and Motta (2016) expound the constraints under which moneylenders or MFIs maximize their utility if they offer a joint liability contract.

Firstly, the breakeven constraint (1) implies that the capital cost \( \rho \) must be smaller or equal to the expected repayment by the borrowers. If the investment project of a borrower is successful, he repays a fix sum \( R \) (individual liability) and, if the project of his partner fails, a joint liability component \( C \). Risky type borrowers (subscript \( r \)) face a low probability of success \( p_r \) and constitute proportion \( \theta \) of the population. The rest \((1 - \theta)\) succeeds with \( p_s > p_r \) and is therefore considered a safe type (subscript \( s \)). It follows:

\[
(1) \, \theta p_r[R_r + C_r(1 - p_r)] + (1 - \theta)p_s[R_s + C_s (1 - p_s)] \geq \rho
\]

Secondly, the participation constraint ensures that the expected payoff \( U_{ii}(R_i, C_i, a) \) for borrowers of risk type \( i \), with \( i = \{r, s\} \), is bigger than landholding \( a \) which they retain without a loan and which is used as a proxy for wealth:

\[
(2) \, U_{ii}(R_i, C_i, a) \geq a
\]

Based on the assumption that loan contracts are not collateralized, the limited liability constraint guarantees that borrowers do not need to repay more than the return of their investment projects \( Q_i(a) \):

\[
(3) \, R_i + C_i \leq Q_i(a)
\]

The incentive-compatibility constraint gives the reason that a borrower of type \( i \) will not choose another contract \( j \):

\[
(4) \, U_{ii}(R_i, C_i) \geq U_{ij}(R_j, C_j)
\]

The authors assume that the MFI maximizes the welfare of its borrowers while an informal lender maximizes his profits (see assumptions 2.1 and 3.1 in Annex 1).

The whole range of assumptions relevant to the breakeven constraint is 4.1, 4.4, 4.6-4.8, 5.1-5.4 and 5.6 in Annex 1.

This constraint is based on assumptions 4.1, 4.3, 4.12 and 5.1 which are described in Annex 1.

Especially in Bangladesh income is often driven by farming which supports the assumption of such a proxy (Zeller et al. 2001).

See 5.2 in Annex 1.

This assumption is also cited in 5.3 in Annex 1. It is further based on assumptions 4.1, 4.2, 4.3, 4.9, 5.4. Moreover assumptions 4.10 and 4.11 are relevant to its further application.

This is based on assumptions 4.13, 4.15 and 5.6 in Annex 1.
Finally, the **ex-post incentive-compatibility constraint** establishes that it would be more expensive for a pair of borrowers to pay the individual liability than to pay joint liabilities. Given that, they will always report the failure of a project.

\[ R_i \geq C_i \]

### 2.1.2 Feasibility of a joint liability pooled contract

On that basis, Mookherjee and Motta (2016) state that a joint liability pooled contract is only feasible if the following condition is true:

\[ p_s Q_s(a) \geq \max \left\{ \frac{p_s(2-p_s)}{\theta p_r(2-p_r)+(1-\theta)p_s(2-p_s)} \rho^l + a, \rho^l \frac{p_s}{\bar{p}} + \beta a \right\}, \]

where \( \beta \equiv \frac{\theta p_r^2+(1-\theta)p_s^2}{p_s \bar{p}} \) and \( \bar{p} \equiv \theta p_r + (1-\theta) p_s \). In the following, I will derive how this inequality ensures that the constraints (1), (2), (3) and (5) are satisfied, i.e. that the contract is feasible.

To begin, the grey colored areas in figure 1 and figure 2 represent the joint liability contracts which fulfill the constraints (3) and (5) while the green colored areas represent those that satisfy (1) and (2).

Fig. 1: The credit market when the MFI is the only lender (Mookherjee and Motta 2016, 195)
Joint liability pooled contracts are feasible if and only if there is an intersection of these two areas. In the figures above, this is the case: all feasible joint liability pooled contracts are represented by BDA in figure 1 and triangle BEF in figure 2. As $\text{PC}_s^{10}$ is always steeper than $\text{ZPC}_{r,s}^{11}$ (see Annex 3 for proof), the green colored area satisfying the participation constraint (2) and the breakeven constraint (1) will always lay above point B, the intersection of $\text{PC}_s$ and $\text{ZPC}_{r,s}$. Therefore, the intersection points of $\text{ZPC}_{r,s}$ with $\text{LLC}_s^{12}$ and $\text{ICC}_{ep}^{13}$ (A and F) must be situated left of the intersection points of $\text{PC}_s$ with $\text{LLC}_s$ and $\text{ICC}_{ep}$, (D and E) respectively, in order to ensure that a part of the green colored area is situated below $\text{LLC}_s$ and $\text{ICC}_{ep}$, thereby satisfying the limited liability constraint (3) and the ex-post incentive-compatibility constraint (5). As the slope of $\text{LLC}_s$ is strictly negative and the slope of $\text{ICC}_{ep}$ strictly positive, this requires $C_A \geq C_D$ and $C_E \geq C_F^{14}$.

As derived in 4.1 and 4.2, $C_A \geq C_D$ is equivalent to $\frac{q_s(a)}{p_s \beta} - \frac{\rho}{p_s \beta} \geq \frac{a}{p_s^2}$. Rearrangement yields

$$p_s Q_s(a) \geq \frac{p_s}{p} \rho + \beta a$$

---

10 $\text{PC}_i$ is the set of joint liability contracts that satisfies participation constraint (2) for a borrower of type $i$ with equality.

11 $\text{ZPC}_i$ is the set of joint liability contracts that satisfies the breakeven constraint (1) for a borrower of type $i$ with equality.

12 $\text{LLC}_i$ is the set of joint liability contracts that satisfies the limited liability constraint (3) for a borrower of type $i$ with equality (i$\neq j$).

13 $\text{ICC}_{ep}$ is the set of joint liability contracts that satisfies the ex-post incentive-compatibility constraint (5) for a borrower of type $i$ with equality.

14 $C_M$ represents the C-coordinate, i.e. the joint liability, of point M, i.e. contract M.
This equals the second term of inequality (6). As explained above, it ensures that there are contracts which not only satisfy constraints (1) and (2), but also the limited liability constraint (3). This corresponds exactly to the its role assigned by Mookherjee and Motta (2016).

As shown in Annex 4.3 and 4.4, \( C_E \geq C_F \) is equivalent to \( \frac{p_s Q_s(a) - a}{p_s(2 - p_s)} \geq \frac{\rho}{p (2 - p_s \beta)} \) which can be rearranged to

\[
(8) \quad p_s Q_s(a) \geq \frac{p_s(2 - p_s)}{p(2 - p_s \beta)} \rho + a = \frac{p_s(2 - p_s)}{\theta p_r(2 - p_r) + (1 - \theta) p_s(2 - p_s)} \rho + a
\]

which equals the first term of (6). Again, its origin complies with Mookherjee and Motta’s designation (2016) as it ensures the existence of a contract that fulfills constraints (1) and (2) as well as the ex-post incentive-compatibility constraint (5).

If both (7) and (8) are valid inequality (6) must also hold. Thus, I verified that (6) ensures the existence of a feasible joint liability pooled contract.

2.2 Analysis of the model’s propositions on informal credit markets\(^{15}\) in isolation

Mookherjee and Motta (2016) assume that the equilibrium of informal credit markets in isolation is reached in a 4-stage game: (1) Informal lenders offer contracts to borrowers from other communities (segments)\(^{16}\) and (2) to borrowers from their own.\(^{17}\) (3) Borrowers agree to one contract or none.\(^{18}\) (4) If the project is successful, the borrower repays the loan.\(^{19}\)

2.2.1 Outreach

The authors propose that all borrowers choose the loan from the lender of their own segment. In line with their reasoning, I prove this in the following:

As moneylenders cannot distinguish the risk types of borrowers from other segments,\(^{20}\) their contracts must break even for the pool of both risk types in the population. Hence, the breakeven constraint is equivalent to \( R^l + C^l (1 - p_s \beta) \geq \frac{\rho^l}{p} \), with \( C^l = 0 \).

---

\(^{15}\) Following Demont (2016), I refer to the “informal credit market” as the traditional moneylending market.

\(^{16}\) One segment represents one community or one village in which borrowers and the lender(s) know each other well (see assumption 1.1 in Annex 1).

\(^{17}\) This is based on the time advantage in the own segment as explained in assumption 2.4 in Annex 1.

\(^{18}\) The authors assume exclusivity of contracts (see 5.1 in Annex 1).

\(^{19}\) See assumptions 5.2 and 5.3 in Annex 1.

\(^{20}\) See 1.1 and 4.5 in Annex 1 for the assumptions of segmentation and asymmetric information.
Within his own segment, a moneylender can discriminate between borrowers of different risk types.

Hence, his breakeven constraint for the safe type implies \( R_s' + C_s'(1 - p_s) \geq \frac{\rho^I_s}{p_s} \). Thus, he can attract safe borrowers by undercutting the external lenders’ offer, since \( p_s \geq \bar{p} \) \(^{22}\) and \( \beta \leq 1 \)\(^{23}\).

For the risky type, reflection of his high default risk in the loan offer requires a repayment of \( R_r' + C_r'(1 - p_r) \geq \frac{\rho^I_r}{p_r} \). As \( p_r \leq \bar{p} \), this would be more expensive than the offer made by external lenders. However, as all safe type borrowers moved to the internal lender, the proportions of risk types in the pooled contracts of external lenders change: \( \bar{p} \) is replaced by \( p_r \)\(^{24}\). Thus, borrowers’ cost of credit charged by external lenders equal those charged by the internal lender. In that case, it is assumed that borrowers prefer the moneylender from their own segment\(^{25}\).

### 2.2.2 Contract choice

Mookherjee and Motta (2016) explain that as long as an internal lender keeps his contract offer below the breakeven constraint of external lenders, he retains market power over safe type borrowers. Hence, he designs a contract that maximizes his payoff. In this context, offering joint liability would not raise profits, which I prove in Annex 5.

As seen above, the market for risky type borrowers involves perfect competition which implies zero profits. Therefore, no change of the contract design can raise their profits.

Given the tiebreaking rule, which assumes that lenders offer individual liability if they are indifferent between this and joint liability\(^{26}\), informal lenders offer only individual liability contracts.

\(^{21}\) As shown in Annex 2, the breakeven constraint for joint liability pooled contracts is (A 2.1) \( \bar{p}[R + C(1 - p_s\beta)] \geq \rho^I \). Division by \( \bar{p} \) leads to the term above. (\( \rho^I \) are the capital cost of an informal lender.)

\(^{22}\) See assumptions 4.4 and 4.8 in Annex 1.

\(^{23}\) See Annex 3 for proof.

\(^{24}\) The external lenders will either know this mechanism theoretically or notice the change by making consistent loss.

\(^{25}\) See 4.16 in Annex 1.

\(^{26}\) See 5.5 in Annex 1.
2.2.3 Interest rate
Consequently, $C_r^l = 0$, and, as lending to risky type borrowers yields zero profit, their interest rate satisfies the breakeven constraint with equality:

$$R_r^l = \frac{\rho^l}{p_r}.$$ (9)

Profits from lending to the safe type are maximized by minimization of borrowers’ payoff. To that end, the participation constraint is satisfied with equality: $p_s Q_s(a) - p_s R_s^l = a = R_s^l = Q_s(a) - \frac{a}{p_s}$. However, if $\frac{\rho^l}{p_r} < Q_s(a) - \frac{a}{p_s}$, borrowers would choose the contract for risky types. This explains the hypothesis from the model

$$R_s^l = \min \left\{ Q_s(a) - \frac{a}{p_s}, \frac{\rho^l}{p_r} \right\}. \quad \text{(10)}$$

2.2.4 Welfare
Borrowers are only better off with a loan if their payoff is bigger than their original wealth: $U_i(R^l_i, a) = p_i Q_i(a) - p_i R^l_i > a$.

For the risky type, $U_r(R^l_r, a) = p_r Q_r(a) - p_r \frac{\rho^l}{p_r} = p_r Q_r(a) - \rho^l$. All projects are assumed to be socially productive which means that the expected return of a project is higher than the sum of its cost (capital and land)$^{27}$. As this is equivalent to $p_r Q_r(a) - \rho^l > a$ risky borrowers are certainly better off with a loan.

For safe type borrowers, there are two possibilities: Either they acquire contracts with $U_r(R^l_s, a) = p_s Q_s(a) - p_s \left[ Q_s(a) - \frac{a}{p_s} \right] = a$, i.e. with wealth equal to autarky. Or the interest rate for risky borrowers is lower than $Q_s(a) - \frac{a}{p_s}$, whereby they could switch to that contract and would be better off.

These results are in line with Mookherjee and Motta’s propositions (2016).

2.3 Analysis of the model’s propositions on the impact of MFI entry
The model assumes that the MFI offers its loans prior to informal lenders at stage 0. Then, three parameters are defined for the derivation of market equilibrium in this 5-stage-game:

---

$^{27}$ See assumption 4.14 in Annex 1.
The first represents the “safe borrowers’ lowest project return that satisfies the limited liability constraint on any joint liability loan offered by the MFI” (Mookherjee and Motta 2016, 195):

\[
\delta \equiv \frac{\beta-1}{\beta} \left\{ \frac{\rho^I}{p_s} - \rho \frac{p_s \left( \theta p_r^2 + (1-\theta) p_s^2 \right)}{\left( \theta p_r^2 + (1-\theta) p_s^2 \right)} \right\}
\]

This term shall be derived in the following (for intermediate steps see Annex 6). First, the limited liability constraint is inserted into the participation constraint. Next, I divide by \( p_s \) to receive the project return instead of the expected return: \( Q_s(a) \geq R_s + \frac{a}{p_s} \). To ensure feasibility, \( R_s \) is then replaced by the breakeven constraint, combined with limited liability. Rearrangement leads to: \( Q_s(a) \geq \frac{\rho}{p} + \frac{a \beta}{p_s} \). Equivalent to the assumption that every project is socially productive, \(^{28} \frac{a}{p_s} \) is replaced by \( Q_s(a) - \frac{\rho^I}{p_s} - x, x > 0 \). Simplification finally yields:

\[
Q_s(a) \geq \frac{\beta}{\beta - 1} \left\{ \frac{\rho^I}{p_s} - \rho \frac{p_s}{\left( \theta p_r^2 + (1-\theta) p_s^2 \right)} + x \right\}
\]

As the lowest project return is required \( \lim_{x \to \infty} x = 0 \), i.e. \( x \) can be omitted. Given that, I end exactly with the reciprocal of \( \delta (11) \) in spite of satisfaction of all conditions to it. Besides, a numerical example given by the authors at a later stage only leads to the results proposed if this reciprocal is applied instead of (11) (see Annex 14). Hence, equation (11) (term (5) in Mookherjee and Motta (2016)) must be corrected to \( \delta^* \equiv \frac{\beta}{\beta - 1} \left\{ \frac{\rho^I}{p_s} - \rho \frac{p_s}{\left( \theta p_r^2 + (1-\theta) p_s^2 \right)} \right\} \) in order to fulfill the role which the authors assign to it.

The second term defined represents the “lowest effective cost of credit at which the MFI can lend to the safe borrowers” (Mookherjee and Motta 2016, 195-196):

\[
\delta_I \equiv \rho \frac{p_s (2-p_s)}{\left( \theta p_r (2-p_r) + (1-\theta) (2-p_r) \right) \rho_s (2-p_s)}
\]

This equals term (8), i.e. the first term of (6) in case \( \alpha = 0 \). This means that the contract satisfies the breakeven constraint with equality. I.e. \( \delta_I \) shows the lowest cost of credit the MFI can offer, while also fulfilling the participation and the ex-post incentive-compatibility constraint. Setting \( \alpha = 0 \) ensures that this holds for all wealth types. Moreover, in case of \( \alpha = 0 \), (13) is always greater than the second term of (6).\(^{29} \)

\(^{28}\) See 4.14 in Annex 1.

\(^{29}\) Since \( p_s < 1 \) and \( \beta < 1 \), \( \frac{2-p_s}{1-p_s} > 1 \Rightarrow \frac{p_s (2-p_s)}{\rho (1-p_s) \rho_s} \rho^I = \frac{p_s (2-p_s)}{\theta p_r (2-p_r) + (1-\theta) (2-p_r) \rho_s (2-p_s)} \rho^I > \frac{\rho^I p_s}{\rho} \). Q.e.d.
implies that the contract also satisfies the limited liability constraint. Thus, (13) corresponds to the role it receives in the model.

Last but not least, the third term represents the “safe borrowers' lowest project return (for given landholding a) that satisfies the limited liability constraint on an MFI loan designed to break-even whenever risky borrowers are the only ones who accept it.” (Mookherjee and Motta 2016, 196):

\[ \gamma(a) \equiv \frac{p_r - \rho}{p_s^2} a - \frac{\rho}{p_r} \]

As in the derivation of (12), participation and limited liability of the safe type are ensured by \( Q_s(a) \geq R_r + \frac{a}{p_s^2} \). However, this time the interest rate is derived to break even for risky types. Inserting this constraint leads to: \( Q_s(a) \geq \frac{\rho}{p_r} + \frac{p_r}{p_s^2} a \) (see Annex 7 for proof). Thus, I demonstrated the validity of equation (14).

On the basis of these three terms, Mookherjee and Motta’s propositions (2016) concerning the results of contract negotiation are examined in the following.

2.3.1 Outreach

First, the authors propose that risky borrowers accept the loan offer of the MFI. This is easily proven: Because of the assumption that the MFI’s capital cost \( \rho \) are lower than the informal moneylender’s \( \rho^I \), the break even constraint is, analogous to 2.2.1, \( R_{MFI} + C(1 - p_s\beta) \geq \frac{\rho_{MFI}}{\rho} \) for the MFI and \( R^I + C(1 - p_s\beta) \geq \frac{\rho^I}{\rho} \) for informal lenders. Thus, the repayment which breaks even for the MFI undercuts that of the informal lender:

\[ \frac{\rho_{MFI}}{\rho} < \frac{\rho^I}{\rho} \]

Second, the model proposes that safe type borrowers choose the loan from their own segments’ lender if (i) \( \rho^I < \delta_I \) or if (ii) \( \rho^I \geq \delta_I \) and \( Q_s(a) < \delta \).

In case (i) the “cost disadvantage of the informal lenders is sufficiently small (\( \rho^I < \delta_I \)), the MFI cannot compete with the informal lenders in lending to any safe type” (Mookherjee and Motta 2016, 195).

As seen in introduction of 2.3, the MFI cannot lend to safe types at lower effective cost than \( \delta_I \). As this corresponds to term (8), calculation of the distance between the
intersection points of PC, and ZPC with ICC analogous yields the lowest effective cost at which an informal moneylender can lend to safe borrowers from his own segment: These equal ρ′ (see Annex 8 for proof). This confirms that he can offer a better contract to safe types than the MFI if ρ′ < δ. With respect to case (ii), I have shown in the beginning of 2.3 that project returns lower than δ∗ cannot satisfy the limited liability constraint on loans from the MFI, i.e. the landholdings of such borrowers are not large enough to be liable for risky borrowers. For the tailor-made contracts with moneylenders from their own segment, however, the limited liability constraint is satisfied as projects are socially productive (see Annex 9 for proof). Given that, borrowers will switch to these loans if their projects do not yield enough return to repay joint liability pooled contracts.

2.3.2 Contract Choice
First, Mookherjee and Motta claim that, even after MFI entry, “Informal lenders always offer individual liability contracts” (2016, 196). This follows the reasoning that was described in 2.2.2 and proven in Annex 5, namely that joint liability would not raise the lenders’ profits.31

Second, the authors announce that the MFI will offer joint liability contracts designed for safe borrowers and individual liability contracts for risky borrowers if it lends to both types, i.e. if cases (i) and (ii) do not apply. Following the argumentation of the model, this is explained in several steps:
To begin, it is stated that “corresponding to any separating pair of contracts satisfying incentive constraints, there exists a pooled contract which leaves both types of borrowers with the same level of utility, and generates the same expected profit for the MFI.” (Mookherjee and Motta 2016). To prove this, I equate the utilities of a pooled and a tailor-made contract for each risk type.32 Insertion of the derived conditions into the formula for profit made through the separating pair of contracts then results in the profit from a pooled contract which is called m2. (see Annex 10 for proof).

30 Emphasis of the author of this thesis.
31 The difference to 2.2.2 is that the MFI provides a better outside option than the informal moneylenders from other segments, as both do not know borrowers’ risk types but the MFI faces lower capital cost.
32 In addition, satisfying the incentive-compatibility constraint ensures that each risk type will choose the contract designed for him in the separating pair, i.e. that tailor-made contracts are feasible even though the MFI does not know the risk types of its borrowers. The condition derived thereby will be relevant in the calculation of liabilities in 2.3.3 and Annex 13.
Next, Mookherjee and Motta (2016) assume that the MFI does not maximize profit but \( V_a \), its borrowers’ expected utility weighted by the welfare importance which it attributes to the respective risk types.\(^{33}\) The authors suggest that maximization of \( V_a \) implies that contract \( m_2 \) satisfies the breakeven constraint with equality. The proof of that in Annex 11 also confirms that not every contract on \( ZP_{r,s} \) is optimal: The best location of \( m_2 \) on \( ZP_{r,s} \) depends not only on the welfare weights, as Mookherjee and Motta state, but also on the respective probabilities of success, for they occur in \( V_a \) with higher potencies than in the breakeven constraint.\(^{34}\)

The next proposition is that the MFI is indifferent between offering \( m_2 \) to both risk types or offering contracts \( m_2 \) and \( m_r \), whereby \( m_r \) entails a lower joint liability \( C \) and a higher individual liability component \( R \) and lies on the same indifference curve for the risky type. Contract \( m_r \) would leave the safe type with a lower utility level (see Annex 12 for proof). Hence, he would stick to \( m_2 \). As the risky type is indifferent between both contracts, Mookherjee and Motta (2016) assume that he chooses the one with the lower joint liability, analogous to the tiebreaking rule of the lenders\(^{35}\). Compared to the separating pair of contracts, both welfare weight and the expected utility levels of both risk types remain the same. Therefore, the MFI is indifferent between these two offers (see Annex 12).\(^{36}\)

Because of the tie-breaking assumption, the model concludes that the MFI will offer the separating pair, as it can then design contract \( m_r \) only with individual liability, i.e. \( C_{mr} = 0 \).

Thus, it is proven that the loans from the MFI are based on individual liability for the risky type and joint liability for the safe type, if it lends to both.

The third case occurs when the **MFI only lends to the risky type borrowers**, which corresponds to case (i) in 2.3.1: \( \rho^l < \delta_l \). In that situation, the tiebreaking rule implies a preference for individual liability.

However, if \( Q_s(a) \geq \gamma(a) \), the MFI would offer loans with joint liability because that would increase the bargaining position of the safe type, as Mookherjee and Motta (2016) outline. The reason for this is that by definition of \( \gamma(a) \) (see 2.3), any higher project return satisfies the limited liability constraint on an MFI loan which is only accepted by the risky

---

\(^{33}\) See assumption 3.1 in Annex 1.

\(^{34}\) The reason for the use of higher potencies is not clarified by the authors and is inconsistent with the formula (Ghatak 2000) applied.

\(^{35}\) See assumption 5.5 in Annex 1.

\(^{36}\) The result in this step would be the same if the MFI’s objective would be its own profit.
type and breaks-even. As the derivation of $\gamma(a)$ showed (see Annex 7), it would also satisfy the other constraints which the MFI faces. In other words, ZPC_{r}^{\text{MFI}} lies within the cut surface of the green and grey colored areas in figures 1 and 2 (see 2.1.2) although above ZPC_{t}^{I}. Thus, the informal moneylender is further constrained in the maximization of profits. Consequently, safe borrowers benefit which increases the welfare objective of the MFI.

2.3.3 Interest and Liability

With respect to the cost of credit, Mookherjee and Motta (2016) state in the beginning that the MFI sets the interest rate $R$ and the joint liability $C$ in a way that generates zero expected profit. As proven in Annex 11, this holds for a pooled contract $m_{2}$. For the pair of contracts $m_{2}$ and $m_{r}$, it is true, too, since they yield the same $V_{a}$ as $m_{2}$.

On the basis of the zero profit condition, I derive the following interest rates (see Annex 13.1): 

$$(15) \quad R_{m2} = \frac{p}{\bar{p}} - C_{m2}(1 - p_{s}\beta)$$

and, as $C_{mr} = 0$: 

$$(16) \quad R_{mr} = \frac{p}{\bar{p}} + C_{m2}(p_{s}\beta - p_{r})$$

Insertion of these rates into $V_{a}$ and maximization leads to (see Annex 13.2). 

$$(17) \quad C_{m2} = \frac{\bar{q}(a)\frac{p}{\bar{p}}(p_{s} + p_{r})}{p_{s}\beta(p_{s} + p_{r}) - (p_{s}^{2} + p_{s}p_{r} + p_{r}^{2})}$$

Next, Mookherjee and Motta outline that “safe borrowers served by the informal lenders pay the same interest rate they used to pay in the absence of the MFI”, if $Q_{s}(a) \leq \gamma(a)$ (2016, 196). The reason for this is that, compared to the situation without MFI, their outside option does not improve if their project return cannot satisfy the limited liability constraint of a contract designed for risky borrowers, as outlined in 2.3.2.

In contrast, the profit maximization of the informal lender is restricted by ZPC_{r}^{\text{MFI}} if $Q_{s}(a) \geq \gamma(a)$. Given the assumption that borrowers prefer lenders of their own segment, the highest individual liability the informal lender can charge equals the price for a contract located on ZPC_{r}^{\text{MFI}}. Since ZPC_{r}^{\text{MFI}} is $p_{r}[R_{r} + C_{r}(1 - p_{r})] = \rho$, the interest rate in this case equals $\frac{p}{p_{r}}$.

37 See 4.16 in Annex 1.
It remains to look at the effects on the **average interest rate on the informal market**, as it was the goal of the model to explain recent empirical findings thereto.

Since risky borrowers originally had to repay more than safe borrowers (see 2.2.3), their move to the MFI decreases the average interest rate on the informal market (Mookherjee and Motta 2016). This would be the final result if all safe borrowers stuck to their informal lender, i.e. in case (i).

In case (ii), however, only those with the lowest landholding remain. Hence, if interest rates are falling in \( a \), MFI entry also has an increasing impact on the average interest rate. If the fraction of safe borrowers in the population was large enough, this effect could exceed the influence of risky borrowers’ move to the MFI. Consequently, the average informal interest rate would rise. (Mookherjee and Motta 2016)

In the paper’s numerical examples \( Q_s(a) = 1 + a^2 \). In case \( Q_s(a) \leq \gamma(a) \), the interest rate remains the same as prior to MFI entry: \( R_s^I = \min \{ Q_s(a) - \frac{a}{p_s} \rho_s^I \} \). Thus, \( R_s^I \) is either independent of \( a \) or it equals \( 1 + a^2 - \frac{a}{p_s} = 1 - a \left( \frac{1}{p_s} - a \right) \). From \( a, p_s < 1 \) follows \( a \left( \frac{1}{p_s} - a \right) > 0 \). That shows that the interest rate would indeed fall with an increasing \( a \). In case \( Q_s(a) \geq \gamma(a) \), \( R_s^I \) would be independent of \( a \) again, as it is \( \frac{\rho_s^I}{p_r} \).

### 2.3.4 Welfare

All borrowers that are served by the MFI are strictly “better off compared with the equilibrium of the informal market without an MFI.” (Mookherjee and Motta 2016). This is clear since their expected project returns remain the same while the cost of their loans have been reduced, as explained in 2.3.1.

Safe borrowers who are served by the informal lender of their segment are strictly better off if their bargaining position has been enhanced, i.e. if \( Q_s(a) \geq \gamma(a) \). Otherwise they gain the same welfare as they did in the market without the MFI, i.e. they are “weakly better off” (Mookherjee and Motta 2016, 196).
3 Evaluation of the model’s explanatory power in comparison with other research

In the beginning of their paper, Mookherjee and Motta (2016) criticize that former theories on the interaction between MFIs and informal moneylenders cannot explain recent empirical research.

For example, Mallick’s findings (2012) are independent of scale diseconomies and collusion, although these have been advanced as possible explanations of rising informal interest rates before (see Floro and Ray 1997; Hoff and Stiglitz 1998; Jain 1999). Therefore, Mookherjee and Motta (2016) do (i) not deem it necessary to account for fix cost and (ii) assume direct MFI lending to borrowers.

Moreover, Berg et al. (2013) find that demand for informal loans decreases with higher MFI coverage which is inconsistent with the theory of crowding in (Jain and Mansuri 2003; McIntosh and Wydick 2005; Kahn and Mookherjee 1998). Hence, Mookherjee and Motta (2016) (iii) see no need to examine the possibility that borrowers demand additional funds from informal lenders and assume exclusivity of contracts instead.

Yet, especially the assumption of exclusive contracts is critical as it is inconsistent with many findings and theories, such as Bell (1990), Siamwalla et al. (1990), Sinha and Matin (1998), Coleman (1999), Zeller et al. (2001), McKernan et al. (2005) and Armendáriz and Morduch (2007). Moreover, Mallick (2012) even explains his results with borrowers’ need for additional funds from informal lenders, but is not able to test this.

All in all, given the neglects (i) to (iii), Mookherjee and Motta’s model (2016) cannot elucidate which conditions and mechanisms in theory prevent the explanations mentioned to hold.

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38 Scale diseconomies imply that the informal lender faces higher average fix cost because of decreased demand through MFI entry (Hoff and Stiglitz 1998; Jain 1999). In fact, Berg et al. (2013) propose this theory as one explanation for their findings, but cannot test it. The second theory suggest that interest rates increase through collusion as this is facilitated through increased availability of credit if formal credit is channelled through informal lenders (Floro and Ray 1997).

39 I.e. increased demand for informal loans because of strict repayment rules and moral hazard (Jain and Mansuri 2003; McIntosh and Wydick 2005; Kahn and Mookherjee 1998). Likewise, Mallick (2012) proposes crowding in as an explanation for his findings, but is not able to test it (Demont 2016).

40 Some authors claim that additional loans from informal lenders are sometimes necessary because of the strict repayment rules of MFIs (Jain and Mansuri 2003). This, however, should not be needed if the MFI satisfies the limited liability constraint, as this entails that repayment is only required if the means are available.

41 The case of MFI lending to informal moneylenders is considered in the extensions. However, their result is that MFI will refrain from doing that if they intend to maximize borrower’s welfare.
Nevertheless, the model does show why the cream-skimming hypothesis is not valid in certain contexts. That theory suggests that informal lenders loose safer clients and that this interest rate increasing composition effect can outweigh the impact of lower demand (competition effect).

Certainly, the reasoning of two opposing forces appears in Mookherjee and Motta’s model (2016) as well. However, in their model both are forces are some kind of composition effects. And, more importantly, the interest rate increasing effect does not stem from MFI’s failed outreach to risky but to poor safe borrowers.

As outreach to the poor and vulnerable represents a crucial issue for MFIs (Yunus and Jolis 1998; Anim et al 2003; BRAC 2015), the next section compares the respective propositions of both models with empirical findings. After that, I will explain how differing assumptions lead to the discrepancies of the two theories with respect to outreach and changes in informal interest rates. Thereby, I will focus on Demont (2016) because he provides the most recent model for cream-skimming and focusses on South Asia just like Mookherjee and Motta (2016) do.42

3.1 Review of empirical research with respect to outreach of MFIs

With respect to poverty targeting, Maitra et al. (2013) find that households with less landholding are more likely to borrow from MFIs than those with more land. This effect is not statistically significant, but consistent with findings made by Zeller et al. (2001).43 Other studies indicating successful outreach to the poor or the poorest are Anim et al. (2003) and Khandker (2003), respectively. However, there have also been studies showing the opposite (Dewan and Somanathan 2007; Ghosh 2013; Dewan and Somanathan 2007).

All in all, literature with respect to poverty targeting is still scarce, as the research interest of MFIs themselves is usually deeper with regard to their impact on the lives of their clients (Ghosh 2013).

As probabilities of success are unobservable, it is hard to account for default risks of borrowers in empirical studies. Therefore, Maitra et al. (2013) use the informal interest rates borrowers as a proxy for their risk type and thereby find that riskier borrowers rather switch to the MFI. This, however, only holds under their assumption that informal lenders

---

42 Another researcher who advanced the cream-skimming hypothesis is Bose (1998). However, he does not refer explicitly to MFIs but to formal moneylenders / banks in general.
43 However, they question that it is a demand-driven effect as some big NGOs in Bangladesh do not give loans to borrowers that own too much land.
can assess the riskiness of borrowers. Similarly, but with the opposite effect, the empirical findings by Demont (2016) depend on his assumptions which are examined below. Hashemi et al. (2001) find that riskier borrowers are often intentionally screened out, a possibility which Ghatak (2000) has theoretically show for risky borrowers whose projects are socially unproductive. Yet, this would be a rather supply-driven effect and does not correspond to the MFI’s failure of attracting risky borrowers which the cream-skimming hypothesis suggests.

In conclusion, empirical research so far is insufficient to evaluate the explanatory power of the cream-skimming theory and Mookherjee and Motta’s opposing model (2016), since even for the latter, the relevant changes in moneylenders’ pool of clients depend on the risk type of the poor borrowers.

3.2 Comparison with the cream-skimming theory of Demont and review of some assumptions’ empirical validity

Both Mookherjee and Motta (2016) and Demont (2016) adopt most assumptions from the widely recognized model of Ghatak (2000). Yet, unlike Demont (2016), Mookherjee and Motta (2016) follow some significant extensions made by Maitra et al. (2013), while they simplify in other important aspects:

In Mookherjee and Motta’s model (2016), (1) informal lenders’ knowledge of the risk types of their own segment’s borrowers44 allows them to keep safe clients after MFI entry, as described in 2.3.1.

Such an informational advantage is also observed by others, such as Duflo (2013) and Stiglitz and Hoff (1990). The latter describe how linkages of trade and lending allow lenders to obtain information about borrowers’ characteristics. However, they also outline that no lender can ever „fully discern the extent of risk of a particular loan“ (1990, 239).

Aleem (1990) found that interest payed for loans of informal lenders is higher than their marginal cost of lending but equals the average cost. This provides evidence for monopolistic competition which can be treated as an indicator for asymmetric information. However, as the scope of this study is geographically and temporally very limited, its

44 See assumption 4.5 in Annex 1.
results do not necessarily have to hold in other contexts. For other markets, there have also been findings which go in the other direction (see Demont 2016). With regard to the empirical findings of Mallick (2012) in Bangladesh, the assumption of monopoly cannot be fully confirmed, too: The number of large landowners, who usually control moneylending activities to a large extent (Bell 1990; Hoff and Stiglitz 1998), differs widely from village to village in rural Bangladesh.

Therefore, Demont (2016) suggests that no lender knows borrowers’ risk types. On that basis, he shows that safe borrowers in his model always switch to the MFI.

Nevertheless, this result only holds because he (2) expects that informal lenders only offer individual liability contracts: Assortative matching then implies that safe borrowers prefer joint liability as its insurance effect allows for lower interest rates (Ghatak 1999). If Demont (2016) supposed flexibility in contracting, as Mookherjee and Motta (2016) do, informal lenders could adapt their contracts and safe borrowers would be indifferent between them and the MFI. However, they would prefer the MFI again if he further introduced a cost advantage of the latter, as in (Mookherjee and Motta 2016).

The loss of safe clients raises interest rates in the informal credit market as it leaves informal lenders with a riskier pool of clients. This in turn would also push the risky borrowers to the MFI. However, Demont (2016) (3) further differs from Mookherjee and Motta (2016) in assuming that the MFI has limited funds. Thereby, it cannot serve all safe borrowers. If the unserved fraction is big enough, the increase in the interest rate will be sufficiently low so that all risky borrowers stay with informal lenders (Demont 2016). Otherwise, the fractions of borrowers unserved by the MFI would be equal for both risk types and consequently the average interest rate in the informal credit market would not change. This means that the model could only explain an increase of the average informal interest rate if MFI funds were low enough. Both Mallick (2012) and Berg et al. (2013), however, observe a stronger effect of increasing informal interest rates with higher levels of MFI coverage. As higher MFI coverage can be seen as an indicator for less limits to MFI funds, I conclude that their findings cannot be well explained by Demont (2016).

---

45 This theory shows that borrowers would not join a group with members of worse risk types than themselves, if the contract involves joint liability and if they know each other’s risk types. The latter assumption is in line with empirical findings in Bangladesh (Zeller et al. 2001). Yet, there have been also other experiences (Ghatak and Guinnane 1999) which imply that this mechanism does not work in every socio-economic context. Nevertheless, an adaptation is not examined, as this assumption is adopted in both models which are compared in this chapter.

46 At this point, following Mookherjee and Motta’s (2016) assumption of a cost advantage for the MFI would raise the maximum limit of funds that would still cause an increase of the informal interest rate.
Nevertheless, the model might account for other contexts; at least they hold with regard to his own empirical testing in Indian villages.

As seen in 2.3, Mookherjee and Motta’s way to resolve the puzzle of increasing interest rates relies on their assumption (4) that landholding is not observable to the econometrician, but to lenders. Consequently, the latter are (5) expected to treat borrowers of different landholdings as separate markets, i.e. to charge varying interest rates. The findings of Maitra et al. (2013) support assumption (5) with respect to informal lenders. MFIs, particularly in Bangladesh, sometimes have several microcredit programs which differ in interest rates and, correspondingly, eligibility criteria with respect to wealth (BRAC 2015; Grameen Bank 2017). Apart from that, however, they usually offer the same contract to all borrowers which is inconsistent with assumption (5). At least this is the case for the largest MFIs in Bangladesh, including the Grameen Bank whose microfinance system has become canonic in the sector (BRAC 2015; Grameen Bank 2017; ASA 2017, 2017; MIX 2017; Duflo 2013). Therefore, the following section sheds some light on possible influences of a corresponding adaptation to the model.

3.3 Consequences of MFIs’ pooling of wealth types

For simplification, I assume that there are two wealth types $a_1$ and types $a_2$, whose welfare weights are $\propto_1$ and $\propto_2$ respectively, with $\propto_1 + \propto_2 = 1$. As $a$ is independent of the risk type, I further suggest that $\propto$ is independent of the risk type and its welfare weight $\lambda$ (which could be changed in another extension of the model). Thus, (for intermediate steps see Annex 15) the objective of the MFI $V_{new}$ is

\[
V_{new} = [\propto_1 \bar{q}(a_1) + \propto_2 \bar{q}(a_2)]\{\lambda p_r + (1 - \lambda)p_s\} - R_{new}\{\lambda p_r^2 + (1 - \lambda)p_s^2\} - C_{new}\{\lambda p_r^3 + (1 - \lambda)p_s^3\}
\]

Analogous to 2.3.3, insertion of $R_{new} = \frac{\nu}{p} - C_{new}(1 - p_s\beta)$ into (18) and maximization. finally yields:

\[
C_{new} = \frac{\propto_1 \bar{q}(a_1) + \propto_2 \bar{q}(a_2) - \frac{\nu}{p}(p_s + p_r)}{p_s\beta(p_s + p_r) - (p_s^2 + p_s p_r + p_r^2)}
\]

47 Usually, programs for poorer borrowers are then supplied with lower interest rates, see for example BRAC (2015) and Grameen Bank (2017).

48 See assumption 4.7 in Annex 1.
From \( a_1 < a_2 \) follows \( \tilde{q}(a_1) < [\alpha_1 \tilde{q}(a_1) + \alpha_2 \tilde{q}(a_2)] < \tilde{q}(a_2) \), as long as project returns are increasing in \( a \). Comparison with (17) implies that \( \tilde{C} \) has decreased for richer borrowers \( (a_2) \) and increased for poorer borrowers \( (a_1) \).

Inserting \( R_{m2/new} = \frac{p}{p} - \frac{C_{m2/new}}{(1 - \rho)} \) into the borrowers payoff function yields:

\[
U_{it}(\frac{C_{m2/new}}{\rho}, a) = \tilde{q}(a) + p_l [\rho \frac{C_{m2/new}}{(1 - \rho)} - \frac{p}{p}]
\]

This shows that the payoff in a unified market would be higher for poorer borrowers and lower for richer borrowers. Thus, pooling markets of different wealth types represents some kind of cross-subsidization. That change of payoffs might lead to a “reversal of MFI participation patterns”, as Mookherjee and Motta (2016, 199) outline in their description of another form of cross-subsidization. Consequently, the average informal interest rate could only increase if informal interest rates would be falling in \( a \). To assess the probability of this, however, further empirical research is needed.
4 Conclusion

The goal of this thesis was to contribute to the explanation and evaluation of existing theory about the impact of microfinance on informal credit markets. To that end, I firstly analyzed and verified the reasoning of the model provided by Mookherjee and Motta (2016). Under the given assumptions, I confirmed most propositions of the model by graphical, mathematical or explanatory derivation and proof. Nevertheless, I also demonstrated how one mathematical term defined in the journal article needs to be corrected. Only thereby the model can explain the increase of the average informal interest rates under the numerical parameters they propose. On the basis of that, it is reasonable to expect that the model could otherwise not account for an increasing informal interest rate at all. Yet, mathematical proof would be needed to confirm this.

In the second section, I compared the model with other theories, particularly the propositions of Demont (2016). This showed that the cream-skimming explanation of spikes in informal interest rates would be more applicable in contexts where lenders have no informational advantage about borrowers’ risk types, landholding is not a factor considered in designing contracts and where MFI funds are severely limited. Particularly because of the latter factor, the model has less explanatory power for recent research from Bangladesh of (Mallick 2012; Berg et al. 2013) than the theory of Mookherjee and Motta (2016).

Overall, the model provided by Mookherjee and Motta (2016) significantly contributes to comprehension of possible effects that certain market characteristics have on the interaction between MFIs and informal lenders.

However, more empirical research is necessary to clarify correlations between risk types, wealth and interest rates of informal moneylenders. On such a basis, the model could also shed further light on the impact of cross-subsidization in microfinance programs.

In addition, the theory does neither account for fix cost nor for competition of numerous MFIs although both have significant impact in South Asia’s microfinance industry.\(^{49,50}\)

\(^{49}\)Especially administrative cost often represent a high proportion in the portfolio of MFIs (McIntosh and Wydick 2005).

\(^{50}\)So far, the model only deals with the entry of one MFI, although the empirical findings they refer to relate to the increase of microfinance program coverage and not the entry of one first MFI (Mallick 2012; Berg et al. 2013). In fact, on average 3.9 MFIs were lending in each of Mallick’s sample villages. McIntosh and Wydick (2005) have shown that competition among MFIs who cross-subsidize is widespread in today’s developing countries and can worsen welfare effects for borrowers. Without cross-subsidization, his
Combined, they could eliminate the cost advantage of MFIs, which is decisive for its outreach and thereby for the average informal interest rate in the model. An extension in this regard could elaborate under which conditions this might be the case. Moreover, it would be a significant step forward to account for factors which influence borrowers’ contract choice apart from the interest rate, since their impact was observed in several studies. Such variable include repayment rules as well as social aspects, as Coleman (1999) and Duflo (2013) outline.

To conclude, more quantitative and qualitative empirical research is needed to examine which assumptions and predictions apply in the context of South Asia and elsewhere. Nevertheless, existing theories such as the model of Mookherjee and Motta (2016) already contribute to a deeper comprehension of different ways and directions in which MFIs impact informal credit markets. This can help such institutions to analyze new markets carefully and to examine all possible side effects in their design of loan contracts. Only by doing so, they will be able to achieve the outreach and welfare effects they aim for.

Word Count: 5919
5 References


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Mallick D (2012) Microfinance and Moneylender Interest Rate: Evidence from Bangladesh


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Note: The proofs in the following rely on assumptions and formulas provided in the model of Mookherjee and Motta (2016), but are derived by the author of this thesis without external help.
1. Assumptions made in the model

Note: Repeated assumptions in the chapters 1 and 2 of Mookherjee and Motta’s paper are summarized at the author’s discretion.

<table>
<thead>
<tr>
<th>Number</th>
<th>Designation from the author of this thesis</th>
<th>Assumptions quoted from Mookherjee and Motta 2016, 193-194</th>
<th>Explanation from Mookherjee and Motta (2016)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Segmentation</td>
<td>“informal credit market is segmented”; “The market is divided into a number of segments, either spatially or on the basis of social relations, wherein residents of each segment know a lot about each other and/or engage in a thick web of social and economic transactions.”</td>
<td>“the assumption of segmentation has been made in many theoretical treatments of informal credit markets in LDCs (Besley, 1994; Mishra, 1994; Basu, 1997; Bardhan and Udry, 1999; Conning and Udry, 2007) and is consistent with empirical evidence (Hoff and Stiglitz, 1990; Yadav et al., 1992; Bell et al., 1997).”</td>
<td>Furthermore consistent with Sianwolla et al. (1990) and Aleem (1990).</td>
</tr>
<tr>
<td>2.1</td>
<td>Profit maximization</td>
<td>“Informal lenders seek to maximize expected profit”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>Flexible about contracting</td>
<td>“Informal lenders react to MFI entry and loan offers by altering the contracts they offer to borrowers.”</td>
<td></td>
<td>Demont (2016) and others assume that informal money lenders always offer individual liability contracts.</td>
</tr>
<tr>
<td>2.3</td>
<td>Monopoly over borrowers of the own segment</td>
<td>“Each segment has one lender and many borrowers.”; Footnote: “If more than one lender, assume they collude perfectly”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.4</td>
<td>Timing advantage in own segment</td>
<td>“The timing captures the additional advantage of dealing with own-segment borrowers, namely the ability to renegotiate the terms of their contracts following an offer from an external lender.”</td>
<td>“it is plausible that lenders can communicate more frequently with members of their own segment, so they can react to offers made by lenders in other segments.”; “Assuming instead that the announcements are simultaneous would</td>
<td>Reasoning is indeed plausible.</td>
</tr>
</tbody>
</table>
not alter our main results substantially but it would complicate the analysis of the equilibrium in the informal market. Namely, the equilibrium would not exist whenever the informal lender is able to offer a set of contracts that satisfy the zero profit condition and also attract both risky and safe borrowers from other segments.”

### 3. Assumptions about the MFI

<table>
<thead>
<tr>
<th></th>
<th>Maximization of borrower’s welfare</th>
<th>Lower capital cost</th>
<th>Timing disadvantage</th>
<th>Only one MFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>“The MFI is a non-profit entity that seeks to maximize the welfare of borrowers, subject to a break-even constraint.”; “The MFI’s objective with respect to borrowers of landholding a is represented by $V_a = \lambda \cdot p_r \cdot U_{rr}(R_{ra}, C_{ra}; a) + (1 - \lambda) \cdot p_s \cdot U_{ss}(R_{sa}, C_{sa}, a)$ where $\lambda \in (0, 1)$ denotes the welfare weight that the MFI assigns to risky borrowers.”</td>
<td>“[the MFI] has access to capital at a lower cost.”</td>
<td>“[…] at stage 0 we allow the MFI to make loan offers.”</td>
<td>Use of singular form throughout the description of the model</td>
</tr>
</tbody>
</table>

| 3.2 | “In general consistent with McIntosh and Wydick (2005), although those further ad fixed costs of administration which are higher for the MFI. No empirical studies have been found on comparison of informal lenders’ and MFI’s overall capital cost. |

| 3.3 | “Nevertheless, the authors assume that the paper “provides an alternative model of interaction between |
MFIs and informal lenders,” note the plural. Moreover, they refer to the empirical findings of Mallick (2012) as motivating their model, although his findings relate to the increase of “microfinance program coverage” and not the entry of one first MFI. McIntosh and Wydick (2005) have shown that competition among MFIs can alter welfare effects for borrowers.

4. Assumptions about borrowers and their projects

<table>
<thead>
<tr>
<th>4.0</th>
<th>Two risk types</th>
<th>This is not mentioned explicitly, but is applied like that throughout the model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Borrowing for investment</td>
<td>“Each borrower is endowed with a risky investment project. The project requires one unit of land (or other inputs) and one unit of capital. Borrowers lack sufficient personal wealth and need to borrow to launch the project.”</td>
</tr>
<tr>
<td>4.2</td>
<td>Observable landholding</td>
<td>“Landholding is perfectly observable to both informal lenders and the MFI, though not the econometrician”</td>
</tr>
<tr>
<td>4.3</td>
<td>Wealth non-collateralizable</td>
<td>“The borrowers are characterized by (i) their (non-collateralizable) wealth $a \geq 0$, which also represents their</td>
</tr>
</tbody>
</table>

Mallick even observes that “Household consumption and repayment of other loans are the two most frequently reported uses of the informal loans.” (2012).

“Since landholding $a$ is observable, the market composed of borrowers with a given landholding $a$ can be treated as an independent market. In what follows we focus on a given $a$, and suppress dependence of parameters on $a$.” (Mookherjee and Motta 2016)
### 4.4
**Outside option under autarky [...]**

“[…] and (ii) their unobservable probabilities of success $p_i$ with $i = \{r, s\}$ and $1 > p_s > p_r > 0$.

It is plausible that default risks are not observable, as also Fischer (2013) notes.

### 4.5
**Asymmetric information about risk type**

“The MFI cannot identify a borrower's risk type, while informal lenders know the risk type of borrowers in their own segment”

“Similar to the theories discussed above”; “Similar results obtain when the lender is better able to enforce loan repayment from safe types within his segment compared to other types or residents of other segments.”

Assumption is consistent with observations made by Duflo (2013). Stiglitz and Hoff, however, outline that no lender can ever „fully discern the extent of risk of a particular loan“ (1990).

### 4.6
**Assortative matching**

“Assuming that borrowers know each other's types, there is assortative matching: safe (resp. risky) borrowers pair up with safe (resp. risky) borrowers.”

“As Ghatak (2000) showed, it is possible for a lender to screen different types using joint liability loans and asking borrowers to form groups.”

The knowledge of risk types among lenders is consistent with empirical findings in Bangladesh (Zeller et al. 2001). Yet, there have been also other experiences with respect to the relationships among borrowers in one group (Fischer 2013; Ghatak and Guinnane 1999).

### 4.7
**Proportion of risk types independent of a**

“We assume that these $p_r, p_s$ are independent of $a$”

„it is easy to extend the analysis when this assumption is dropped.”

Inconsistent with McIntosh and Wydick (2005).

### 4.8
**Return = Q_i(a) or 0**

“The return of a project […] $Q_i(a)$ if successful, and 0 if not, where $Q_i(a) > 0$; $i = r, s$.”

Strong simplification, however, the model could be extended to account for a continuum which would probably have no significant implications on the results.

### 4.9
**Increasing returns in a**

“Project returns are increasing in $a$.”

“This reflects reduction in distortions associated with tenancy, ranging from inferior quality of leased in land to...”
<table>
<thead>
<tr>
<th>Section</th>
<th>Topic</th>
<th>Formula</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.11</td>
<td>Equality of expected returns</td>
<td>( p_s Q_s(a) = p_r Q_r(a) \equiv \overline{Q}(a) )</td>
<td>“To simplify the exposition” Consistent with Stiglitz and Weiss (1981), but inconsistent with Meza and Webb (1987) who assume that the project return of both risk types is the same. As Ghatak shows, “These two distinct notions of riskiness lead to very different equilibrium outcomes in the credit market.” (2000). In the model of this paper, however, I expect that it would only complicate the formulas derived in Annex 11 and 13.2, but not change the main results of the model.</td>
</tr>
<tr>
<td>4.13</td>
<td>maximization of ( \overline{Q}(a) )</td>
<td>“[…] and maximize expected returns.”</td>
<td>Both payoff and return based decision-making with respect to credit inconsistent with a study from South Africa (Duflo 2013)</td>
</tr>
<tr>
<td>4.14</td>
<td>Socially productive projects</td>
<td>“All projects are socially productive in the sense that ( p_i Q_i(a) &gt; \rho^i + a ) for all ( a ) and ( i = {r, s} ).”</td>
<td>Not plausible as ( p_i ) unobservable.</td>
</tr>
<tr>
<td>4.15</td>
<td>Preference for lowest interest rate</td>
<td>“Using individual liability contracts […] both types would opt for the loan with the lowest interest rate”</td>
<td>Inconsistent with empirical findings (Duflo 2013), but usual assumption in economic models.</td>
</tr>
<tr>
<td>4.16</td>
<td>Preference for own segment lender</td>
<td>“[…] borrowers prefer to be served by their own-segment lender whenever they are indifferent and the latter makes positive”</td>
<td>“This assumption is not substantive, and simplifies the exposition.”</td>
</tr>
</tbody>
</table>
## 5. Assumptions about lending

<table>
<thead>
<tr>
<th></th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td><strong>Exclusive contracts</strong></td>
</tr>
<tr>
<td></td>
<td>“Lending contracts are exclusive, […]. Hence MFI borrowing leads to crowding out of loans from informal lenders.”</td>
</tr>
<tr>
<td></td>
<td>“consistent with the findings of Berg et al. (2013)”</td>
</tr>
<tr>
<td></td>
<td>Inconsistent findings and theories of Bell (1990), Siamwalla et al. (1990), Sinha and Matín (1998), Coleman (1999), Zeller et al. (2001), McKernan et al. (2005), Armendáriz and Morduch (2007)</td>
</tr>
<tr>
<td>5.2</td>
<td><strong>No collateral</strong></td>
</tr>
<tr>
<td></td>
<td>“Loan contracts offered by informal lenders or the MFI are not collateralized and can involve either individual liability or joint liability.”</td>
</tr>
<tr>
<td>5.3</td>
<td><strong>Limited liability</strong></td>
</tr>
<tr>
<td></td>
<td>“Borrowers repay whenever they have the means to do so, i.e., consistent with limited liability.”</td>
</tr>
<tr>
<td>5.4</td>
<td><strong>Joint liability: Groups of two</strong></td>
</tr>
<tr>
<td></td>
<td>“[Joint liability] involves asking the borrowers to form groups of two, and offering an individual liability component R and a joint liability component C.”</td>
</tr>
<tr>
<td></td>
<td>“Most models of group lending examine the case of two-person groups. See Ahlin (2013) [see Ahlin (2015)] and Maitra et al. (2013) for models of group lending under adverse selection with group size greater than two.”</td>
</tr>
<tr>
<td></td>
<td>Usually groups consist of more than two borrowers (Ghatak and Guinnane 1999). However, I propose that the model could be extended in that way without significant changes in results.</td>
</tr>
<tr>
<td>5.5</td>
<td><strong>Tiebreaking rule</strong></td>
</tr>
<tr>
<td></td>
<td>“lenders offer individual liability loans if they earn the same expected profit with joint liability loans.”</td>
</tr>
<tr>
<td>5.6</td>
<td><strong>Fixed loan size</strong></td>
</tr>
<tr>
<td></td>
<td>“Given the loan size is fixed, it is impossible for the MFI to screen different types using individual liability contracts.”</td>
</tr>
<tr>
<td>5.7</td>
<td><strong>No cross-subsidization</strong></td>
</tr>
<tr>
<td></td>
<td>“We initially suppose that the MFI has no redistributive objectives across borrowers of diverse landholdings.”</td>
</tr>
<tr>
<td></td>
<td>Altered in chapter on extensions.</td>
</tr>
</tbody>
</table>
2. Derivation of ZPC_{r,s}, PC_s, ICC_{ep} and LLC_s

To obtain figures of the model, I determine firstly the functions of the given constraints:

**ZPC_{r,s}...**

...represents breakeven for a *pooled* contract. Therefore, the lender does not distinguish
between the risky and the safe type: \( R_r = R_s \) and \( C_r = C_s \). Thus, ZPC_{r,s} is given by:

\[
\theta p_r [R + C(1 - p_r)] + (1 - \theta) p_s [R + C(1 - p_s)] = \rho \]

\[
R [\theta p_r + (1 - \theta) p_s] + C [\theta p_r (1 - p_r) + (1 - \theta) p_s (1 - p_s)] = \rho
\]

As \( \bar{p} \equiv \theta p_r + (1 - \theta)p_s \) this equals:

\[
R \bar{p} + C \{ \bar{p} - [\theta p_r^2 + (1 - \theta) p_s^2] \} = \rho
\]

\[
\beta \equiv \frac{\theta p_r^2 + (1 - \theta) p_s^2}{p_s \bar{p}} \text{ leads to:}
\]

\[
R \bar{p} + C(\bar{p} - p_s \bar{p} \beta) = \rho
\]

\[
= (A 2.1) \quad \bar{p} [R + C(1 - p_s \beta)] = \rho
\]

Rearrangement leads to

\[
(A 2.2) \quad C_{ZPC_{r,s}}(R) = \frac{\rho}{\bar{p}(1 - p_s \beta)} - \frac{R}{1 - p_s \beta}
\]

**PC_s...**

is extended by inserting \( U_{it}(R_i, C_i, a) = p_i Q_i(a) - [p_i R + p_i (1 - p_i) C] \) (Mookherjee and Motta 2016, 193):

\[
(A 2.3) \quad p_s Q_s(a) - p_s [R + (1 - p_s) C] = a
\]

\[
=> \quad p_s Q_s(a) = p_s [R + (1 - p_s) C] + a
\]

Rearrangement leads to

\[
(A 2.4) \quad C_{PC_s}(R) = \frac{Q_s(a)}{1 - p_s} - \frac{a}{p_s (1 - p_s)} - \frac{R}{1 - p_s}
\]

**ICC_{ep}...**

\[
(A 2.5) \quad C_{ICC_{ep}}(R) = R
\]

**LLC_s...**

\[
=> (A 2.6) \quad C_{LLC_s}(R) = Q_s(a) - R
\]

To obtain figure 2, I adopt the values which Mookherjee and Motta (2016) use for figure 1
\( (Q_s(a) = 1 + a^2; \ p_r = 0.4; \ p_s = 0.7; \ \theta = 0.6) \), except that I increase \( a \) from 0.7 to 0.95,
\( \rho \) from 0.6 to 0.62, \( \pi \) from 0.45 to 0.5 (the authors normalize the outside option to \( a-\pi \)).
Then, I solve the functions, derived above, in Microsoft Excel, discretizing $R$ and $C$, and receive the following coordinates:

<table>
<thead>
<tr>
<th>$Qs(a)$</th>
<th>$\theta$</th>
<th>$p^ -$</th>
<th>$R$</th>
<th>$C_{\text{ZPC}_{r,s}}$</th>
<th>$C_{\text{PC}_s}$</th>
<th>$C_{\text{LLC}_s}$</th>
<th>$C_{\text{ICC}_s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>0</td>
<td>2.7193</td>
<td>3.9607</td>
<td>1.903</td>
<td>0</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>0.15</td>
<td>2.3772</td>
<td>3.4607</td>
<td>1.753</td>
<td>0.15</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>0.3</td>
<td>2.0351</td>
<td>2.9607</td>
<td>1.603</td>
<td>0.3</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>0.45</td>
<td>1.693</td>
<td>2.4607</td>
<td>1.453</td>
<td>0.45</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>0.6</td>
<td>1.3509</td>
<td>1.9607</td>
<td>1.303</td>
<td>0.6</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>0.75</td>
<td>1.0088</td>
<td>1.4607</td>
<td>1.153</td>
<td>0.75</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>0.9</td>
<td>0.6667</td>
<td>0.9607</td>
<td>1.003</td>
<td>0.9</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>1.05</td>
<td>0.3246</td>
<td>0.4607</td>
<td>0.853</td>
<td>1.05</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>1.2</td>
<td>-0.018</td>
<td>-0.0393</td>
<td>0.703</td>
<td>1.2</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>1.35</td>
<td>-0.36</td>
<td>-0.5393</td>
<td>0.553</td>
<td>1.35</td>
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<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>1.5</td>
<td>-0.702</td>
<td>-1.0393</td>
<td>0.403</td>
<td>1.5</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>1.65</td>
<td>-1.044</td>
<td>-1.5393</td>
<td>0.253</td>
<td>1.65</td>
</tr>
<tr>
<td>1.903</td>
<td>0.8022</td>
<td>0.52</td>
<td>1.8</td>
<td>-1.386</td>
<td>-2.0393</td>
<td>0.103</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Using Microsoft Excel, these figures are then illustrated graphically which yields figure 2.

3. The slopes of $PC_s$ and $ZPC_{r,s}$

Given the functions (A 2.2) and (A 2.4), the slope of $ZPC_{r,s}$ is $\beta = \frac{1-p^s \theta}{1-p^s}$ and the slope of $PC_s$ is $\beta = \frac{\partial p^2_s + (1-\theta)p^2_s}{p^s \bar{p}}$.

From $1 \geq p_r, p_s, \theta \geq 0$ (see Annex 1) follows that $\beta = \frac{\partial p^2_s + (1-\theta)p^2_s}{p^s \bar{p}} = \frac{\partial p^2_r + (1-\theta)p^2_r}{p^r \bar{p}} \geq 0$.

In addition $\beta = \frac{\partial p^2_s + (1-\theta)p^2_s}{\partial p_r p^r_s + (1-\theta)p^2_s} \leq 1$ because $p_s > p_r$ (see Annex 1) and thereby $\partial p_r p^s + (1-\theta)p^2_s \geq \partial p^2_r + (1-\theta)p^2_s$.

Given that $\beta p_s \leq p_s \implies 1 - \beta p_s \geq 1 - p_s \implies \frac{1}{1-p_s} \leq \frac{1}{1-p_s}$. This proves that $PC_s$ is steeper than $ZPC_{r,s}$.

4. Joint liability-coordinates (C) of points A, D, E anf F

4.1 Point A

...is the intersection of $ZPC_{r,s}$ and $LLC_s$.

$ZPC_{r,s}$ (A 2.1):

$\bar{p}[R + C(1-p_s \beta)] = p$

Rearrangement of $LLC_s$ ($R + C = Qs(a)$) leads to:

$$ R = Q_s(a) - C $$

In order to get the intersection of $ZPC_{r,s}$ and $LLC_s$ we replace $R$ in $ZPC_{r,s}$ by $R$ from $LLC_s$: 38
\[ \bar{p}[Q_s(a) - C_A + C_A(1 - p_s\beta)] = \rho \]
\[ = \bar{p}[Q_s(a) + \beta p_s C_A] = \rho \]
\[ \Rightarrow C_A = \frac{Q_s(a)}{p_s\beta} - \frac{\rho}{p_s\bar{p}\beta} \]

4.2 Point D

…is the intersection of PC_s and LLC_s.

PC_s (A 2.3):
\[ p_s Q_s(a) - p_s[R + (1 - p_s)C] = a \]
\[ \Rightarrow p_s Q_s(a) = p_s[R + (1 - p_s)C] + a \]

Replacing R by \( R = Q_s(a) - C \) from LLC_s yields:
\[ p_s Q_s(a) = p_s[Q_s(a) - C_D + (1 - p_s)C_D] + a \]
\[ \Rightarrow 0 = -p_s^2 C_D + a \]
\[ \Rightarrow C_D = \frac{a}{p_s^2} \]

4.3 Point E

…is the intersection of PC_s and ICC_{ep}.

As seen in (A 2.3) PC_s leads to:
\[ p_s Q_s(a) = p_s[R + (1 - p_s)C] + a \]

Replacing R by \( R = C \) from ICC_{ep} yields:
\[ p_s Q_s(a) = p_s(2 - p_s)C_E + a \]
\[ \Rightarrow C_E = \frac{p_s Q_s(a) - a}{p_s(2 - p_s)} \]

4.4 Point F

…is the intersection of ZPC_{r,s} and ICC_{ep}.

ZPC_{r,s} (A 2.1):
\[ \bar{p}[R + C(1 - p_s\beta)] = \rho \]

Replacing R by \( R = C \) from ICC_{ep} yields:
\[ \bar{p} C_F(2 - \beta p_s) = \rho \]
\[ \Rightarrow C_F = \frac{\rho}{\bar{p}(2 - p_s\beta)} \]

5. Profits made by joint liability (jl) and individual liability (il) contracts offered by informal lenders within their own segment

As Mookherjee and Motta (2016) assume that contracts are exclusive (see assumption 5.1 in Annex 1), loan demand per borrower will either be 1 or 0. If the participation constraint
(2) is satisfied, a safe borrower will accept the loan offered by the informal lender in his own segment of the market (as explained in 2.2).

The profit made by the lender is expected repayment (ER) minus capital cost.
\[ \pi_k = ER_k - \rho \]
with \( k = \{il, jl\} \)

Satisfaction of (2) requires \( p_s Q_s(a) - ER_k \geq a \)

Benefitting from his market power, the informal lender will maximize profits by maximizing \( ER \) so far that it satisfies the participation constraint with equality and that it remains below the contract set up for safe and risky types together by external lenders.

Given that these constraints are equal both for joint and individual liability, the maximized \( ER \) will be the same regardless of the contract. It will only take different forms:

\[
ER_{jl} = p_s [R_{jl} + (1 - p_s)C] \\
ER_{il} = p_s R_{il} \\
\Rightarrow R_{il} = R_{jl} + (1 - p_s)C
\]

6. The safe borrowers' lowest project return satisfying the limited liability constraint on any joint liability loan offered by the MFI

Participation constraint for a joint liability contract from (A 2.3):
\[ p_s Q_s(a) \geq p_s [R + (1 - p_s)C] + a \]

Insertion of the limited liability constraint \( C_{LLC_s}(R) \geq Q_s(a) - R \) is possible:
\[
p_s Q_s(a) \geq p_s [R + (1 - p_s)C] + a \geq p_s [R + (1 - p_s)(Q_s(a) - R)] + a \\
= p_s [p_s R + (1 - p_s)Q_s(a)] + a
\]

Rearrangement leads to:
\[ p_s^2 Q_s(a) \geq p_s^2 R + a \]

Division by \( p_s^2 > 0 \) yields:
\[
(A 6.1) \quad Q_s(a) \geq R_s + \frac{a}{p_s^2}
\]

To be offered by the MFI, the contract has to satisfy the MFI’s breakeven constraint which is (A 2.1):
\[ \bar{p}[R + C(1 - \beta p_s)] \geq \rho \]

As \( C_{LLC_s}(R) \geq Q_s(a) - R \) results in \( \bar{p}[R + (Q_s(a) - R)(1 - p_s\beta)] \geq \bar{p}[R + C(1 - p_s\beta)] \)

insertion of the limited liability constraint is again possible:
\[
\bar{p}[R + (Q_s(a) - R)(1 - p_s\beta)] = \bar{p}[R p_s\beta + Q_s(a)(1 - p_s\beta)] \geq \rho \\
\Rightarrow (A 6.2) \quad R \geq \frac{\rho}{p_s\beta} - \frac{Q_s(a)}{p_s\beta} + Q_s(a)
\]
This is inserted into (A 6.1):

\[ Q_s(a) \geq \frac{\rho}{p_s \beta} - \frac{Q_s(a)}{p_s \beta} + Q_s(a) + \frac{a}{p_s^2} \]

\[ \Rightarrow \frac{Q_s(a)}{p_s \beta} \geq \frac{\rho}{p_s \beta} + \frac{a}{p_s^2} \]

(A 6.3)

\[ \Rightarrow Q_s(a) \geq \frac{\rho}{p} + \frac{a \beta}{p} \]

Assumption 4.14 says: \( p_s Q_s(a) > \rho' + a \Rightarrow p_s Q_s(a) - x = \rho' + a \) with \( x > 0 \).

\[ \Rightarrow Q_s(a) - \frac{\rho'}{p_s} - x = \frac{a}{p_s} \]

Insertion into (A 6.3) leads to:

\[ Q_s(a) \geq \frac{\rho}{p} + \beta \left[ Q_s(a) - \frac{\rho'}{p_s} - x \right] \]

\[ \Rightarrow (1 - \beta) Q_s(a) \geq \frac{\rho}{p} + \beta \left[ -\frac{\rho'}{p_s} - x \right] = -\beta \left\{ -\frac{\rho}{p \beta} + \frac{\rho'}{p_s} + x \right\} \]

\[ \Rightarrow Q_s(a) \geq \frac{-\beta}{1 - \beta} \left\{ -\frac{\rho}{p \beta} + \frac{\rho'}{p_s} + x \right\} = \frac{\beta}{1 - \beta} \left( \frac{\rho'}{p_s} - \rho \frac{p_s}{p} (\theta p_r^2 + (1 - \theta)p_s^2) + x \right) \]

7. The safe borrowers' lowest project return satisfying the limited liability constraint on a joint liability loan designed for risky borrowers

As derived in Annex 6 (but this time with an interest rate designed for a risky borrower), satisfaction of the participation and limited liability constraint is ensured by:

(A 7.1) \[ Q_s(a) \geq R_r + \frac{a}{p_s^2} \]

If only risky borrowers accept the contract the breakeven constraint is:

\[ p_r [R_r + C_r (1 - p_r)] \geq \rho \]

Insertion of the limited liability constraint \((-C_{LLC}(R) \leq R_r - Q_s(a))\) leads to:

\[ p_r [R_r - (-C_r)(1 - p_r)] \geq p_r [R_r - (R_r - Q_s(a))(1 - p_r)] \]

\[ = p_r [p_r R_r + Q_s(a)(1 - p_r)] \geq \rho \]

\[ \Rightarrow R_r \geq -\frac{Q_s(a)(1 - p_r)}{p_r} + \frac{\rho}{p_r^2} \]

This is inserted into (A 6.1):

\[ Q_s(a) \geq R_r + \frac{a}{p_s^2} \geq -\frac{Q_s(a)(1 - p_r)}{p_r} + \frac{\rho}{p_r^2} + \frac{a}{p_s^2} \]

Rearrangement results in:
\[ Q_s(a) \left[ 1 + \frac{(1 - p_r)}{p_r} \right] = \frac{Q_s(a)}{p_r} \geq \frac{\rho}{p_r^2} + \frac{a}{p_s^2} \]

\[ \Rightarrow Q_s(a) \geq \frac{\rho}{p_r} + \frac{p_r}{p_s^2}a \]

q.e.d.

8. The lowest effective cost of credit at which the informal lender can lend to the safe borrowers of his own segment

Analogous to (A 4.3), point E is the intersection of PC\(_s\) and ICC\(_{ep}\). The only difference is that we are not looking at a pooled contract but a tailor-made one for safe borrowers. PC\(_s\) leads to:

\[ p_s Q_s(a) = p_s[R_s + (1 - p_s)C_s] + a \]

Replacing \( R \) by \( R = C_s \) from ICC\(_{ep}\) yields:

\[ p_s Q_s(a) = p_s(2 - p_s)C_E^l + a \]

\[ \Rightarrow C_E^l = \frac{p_s Q_s(a) - a}{p_s(2 - p_s)} \]

Analogous to (A 4.4), point F the intersection of ZPC\(_s\) and ICC\(_{ep}\). The difference again is that we are not looking at a pooled contract but a tailor-made one for safe borrowers. Hence:

ZPC\(_s\):

\[ p_s[R_s + C_s(1 - p_s)] = \rho^l \]

Replacing \( R \) by \( R = C \) from ICC\(_{ep}\) yields:

\[ p_s C_F^l(2 - p_s) = \rho^l \]

\[ \Rightarrow C_F^l = \frac{\rho^l}{p_s(2 - p_s)} \]

The slope of PCs is not steeper but equal to the slope of ZPCs:

\[ C_{ZPCs}(R_s) = \frac{\rho^l \rho}{p_s(1-p_s)} - \frac{R_s}{1-p_s} \]

and \( C_{PCs}(R_s) = \frac{Q_s(a)}{(1-p_s)} - \frac{a}{p_s(1-p_s)} - \frac{R_s}{1-p_s} \)

As the slope of ICC\(_{ep}\) is strictly positive, we can still demand that \( C_E^l \geq C_F^l \) in order to ensure that there is a feasible contract for safe types (as in 3.2) although it is not a pooled one. Hence:

\[ \frac{p_s Q_s(a) - a}{p_s(2 - p_s)} \geq \frac{\rho^l}{p_s(2 - p_s)} \]

Which is equivalent to:
\[ p_s Q_s(a) - a \geq \rho^l \]

As for \( \delta_1 \), we assume that \( a = 0 \):

\[ p_s Q_s(a) \geq \rho^l \]

Thus, \( \rho^l \) is the lowest effective cost of credit at which the MFI can lend to the safe borrowers.

9. **Lowest project return that satisfies the limited liability constraint of any contract offered by the informal lender in his own segment**

As derived in Annex 7 (but this time with an interest rate designed for a safe borrower) satisfaction of the participation and limited liability constraint is ensured by:

\[
(A 9.1) \\
Q_s(a) \geq R_s + \frac{a}{p_s^2}
\]

As the contract is only offered to safe borrowers the breakeven constraint is:

\[ p_s[R_s + C_s(1 - p_r)] \geq \rho^l \]

Insertion of the limited liability constraint \((-C_{LLL}(R_s) \leq R_s - Q_s(a))\) leads to:

\[ p_s[R_s - (-C_s)(1 - p_s)] \geq p_s[R_s - (R_s - Q_s(a))(1 - p_s)] = p_s[R_s + Q_s(a)(1 - p_s)] \geq \rho^l \]

\[ \Rightarrow R_s \geq - \frac{Q_s(a)(1 - p_s)}{p_s} + \frac{\rho^l}{p_s^2} \]

This is inserted into \( (A 6.1) \):

\[ Q_s(a) \geq R_s + \frac{a}{p_s^2} \geq - \frac{Q_s(a)(1 - p_s)}{p_s} + \frac{\rho^l}{p_s^2} + \frac{a}{p_s^2} \]

Rearrangement results in:

\[ Q_s(a) \left[ 1 + \frac{(1 - p_s)}{p_s} \right] = \frac{Q_s(a)}{p_s} \geq \frac{\rho^l}{p_s^2} + \frac{a}{p_s^2} \]

\[ \Rightarrow Q_s(a) \geq \frac{\rho^l}{p_s} + \frac{a}{p_s} \]

10. **Equality of the MFI’s profit through a pair of joint liability contracts and a pooled contract with the same utility for risk types**

1. A pair of contracts satisfies the incentive compatibility constraints of both risk types if:

   I. \( U_{ss}(R_s, C_s) \geq U_{ss}(R_r, C_r) \)

   \[ \Rightarrow p_s Q_s(a) - p_s[R_s + (1 - p_s)C_s] \geq p_s Q_s(a) - p_s[R_r + (1 - p_s)C_r] \]

   \[ \Rightarrow (A 10.1) \quad R_s + (1 - p_s)C_s \leq R_r + (1 - p_s)C_r \]
II. \[ U_{rr}(R_r, C_r) \geq U_{rr}(R_s, C_s) \]

\[ => p_r Q_r(a) - p_r[R_r + (1 - p_r)C_r] \geq p_r Q_r(a) - p_r[R_s + (1 - p_r)C_s] \]

\[ => (A\ 10.2) \quad R_r + (1 - p_r)C_r \leq R_s + (1 - p_r)C_s \]

This ensures that each risk type chooses the loan designed for him and is the condition for the following to hold.

2. The pooled contract is required to leave borrowers with the same level of utility. As the assumption of assortative matching (4.6 in Annex 1) also holds for the pooled contract, the following can be derived:

I. \[ U_{ss}(R_p, C_p) = U_{ss}(R_s, C_s) \]

\[ => p_s Q_s(a) - p_s[R_p + (1 - p_s)C_p] = p_s Q_s(a) - p_s[R_s + (1 - p_s)C_s] \]

\[ => (A\ 10.3) \quad R_p + (1 - p_s)C_p = R_s + (1 - p_s)C_s \]

II. \[ U_{rr}(R_p, C_p) = U_{rr}(R_r, C_r), \text{ analogously} \]

\[ => (A\ 10.4) \quad R_p + (1 - p_r)C_p = R_r + (1 - p_r)C_r \]

3. Moreover, the pooled contract is required to make the same profit as was made through separating pair of contracts:

\[ \pi_{pooled} = \pi_r + \pi_s \]

\[ \pi_s + \pi_r \text{ must be the sum of the differences of expected repayment and capital cost in both contracts, weighted with the proportions of the respective borrowers in the population:} \]

\[ \pi_r + \pi_s = \theta^* [p_r[R_r + (1 - p_r)C_r] - \rho] + (1 - \theta^*) [p_s[R_s + (1 - p_s)C_s] - \rho] \]

In the same way, the profit of the MFI made through a pooled contract is the difference the expected repayment and the capital cost. The expected repayment of a pooled contract was derived in Annex 2 (see (A 2.1)).

\[ \pi_{pooled} = \bar{p} [R_p + C_p (1 - p_s \beta)] - \rho \]

It follows:

\[ \bar{p} [R_p + C_p (1 - p_s \beta)] - \rho = \theta^* p_r [R_r + (1 - p_r)C_r] + (1 - \theta^*) p_s [R_s + (1 - p_s)C_s] - \rho \]

From (A 10.3) and (A 10.4) follows:

\[ \bar{p} [R_p + C_p (1 - p_s \beta)] = \theta^* p_r [R_p + (1 - p_r)C_p] + (1 - \theta^*) p_s [R_p + (1 - p_s)C_p] \]

\[ = [\theta^* p_r + (1 - \theta^*) p_s] * [R_p + C_p] - C_p [\theta p_r^2 + (1 - \theta) p_s^2] \]

\[ = \bar{p} * [R_p + C_p] - C_p [\beta \bar{p} p_s] = \bar{p} [R_p + C_p (1 - p_s \beta)] \]

q.e.d
11. Proof of the zero profit condition in case of a pooled contract

Following assumption 3.1 in Annex 1, the weighted sum of expected utilities $V_a$ for contract $m_2$ (and, with $R$ and $C$, also all other pooled contracts) is:

$$V_a = \lambda p_r U_{rr}(R_{m_2}, C_{m_2}; a) + (1 - \lambda) p_s U_{ss}(R_{m_2}, C_{m_2}; a)$$

$$= \lambda p_r \{p_r Q_r(a) - p_r [R_{m_2} + (1 - p_r) C_{m_2}]) + (1 - \lambda) p_s \{p_s Q_s(a) - p_s [R_{m_2} + (1 - p_s) C_{m_2}])\}$$

Replacement of $p_r Q_r(a)$ and $p_s Q_s(a)$ by $\ddot{Q}(a)$ (see assumption 4.11 in Annex 1) and rearrangement leads to:

$$(A\ 11.1)\ V_a = \ddot{Q}(a)[\lambda p_r + (1 - \lambda)p_s] - R_{m_2}[\lambda p_r^2 + (1 - \lambda)p_s^2]$$

$$- C_{m_2}[(\lambda p_r^2 + (1 - \lambda)p_s^2] - [\lambda p_r^3 + (1 - \lambda)p_s^3])$$

With $\bar{p_V} \equiv [\lambda p_r + (1 - \lambda)p_s]$, $\bar{\bar{p}} \equiv [\lambda p_r^2 + (1 - \lambda)p_s^2]$ and $\varepsilon_V \equiv \frac{\lambda p_r^2 + (1 - \lambda)p_s^2}{p_s \bar{p} \bar{V}}$ this can be rearranged to:

$$(A\ 11.2)\ C_{m_2} = \frac{\bar{p_V}\ddot{Q}(a) - V_a}{\bar{p} \ddot{V}(1 - p_s \varepsilon_V)} - \frac{R_{m_2}}{(1 - p_s \varepsilon_V)}$$

$V_{V_a}$ is the set of all joint liability pooled contracts that satisfies this term for a given $V_a$. Thus, every increase of $V_a$ will move the $V_{V_a}$ downwards, as $0 < \lambda, p_r, p_s < 1$ and $p_s > p_r$ lead to $\varepsilon_V < 1$ and thereby to $\bar{p} \ddot{V}(1 - p_s \varepsilon_V) > 0$. Thus, the contract with the highest $V_a$ for which the MFI breaks even will always be the intersection of $V_{V_a}$ and $ZPC_{r,s}$. That implies that the breakeven constraint is satisfied with equality, i.e. the MFI makes zero profit.

This is visualized in the following figures which demonstrate that not every contract that satisfies the breakeven constraint with equality and also fulfills the other constraints (2), (3) and (5), maximizes $V_a$ as the slope of $V_{V_a}$ is usually not equal to the slope of $ZPC_{r,s}$. The intersection of $V_{V_a *}$ with $ZPC_{r,s}$ in each figure represents the contract which satisfies all given constraints with the highest $V_a$. $V_{V_a_{lowV}}$ includes contracts that satisfy all given constraints but entails a lower $V_a$ than $V_{V_a *}$. The reverse is true for $V_{V_a_{highV}}$. 

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Figure 1: \( VV_a \) is steeper than \( ZPC_{r,s} \)

Figure 2: The slopes of \( VV_a \) and \( ZPC_{r,s} \) are equal.
12. Indifference of the MFI between offering a pooled contract or a separating pair of contracts on the same indifference curve

The proof is undertaken similarly to the one in Annex 10:

1. Contract $m_r$ lies on the same indifference curve as contract $m_2$. This satisfies the incentive compatibility constraint with equality and is the case if:

$$U_{rr}(R_{mr}, C_{mr}, a) = U_{rr}(R_{m2}, C_{m2}, a)$$

$$=> p_r Q_r(a) - p_r[R_{mr} + (1 - p_r)C_{mr}] = p_r Q_r(a) - p_r[R_{m2} + (1 - p_r)C_{m2}]$$

$$=> (A \ 12.1) \quad R_{mr} + (1 - p_r)C_{mr} = R_{m2} + (1 - p_r)C_{m2}$$

$$=> (A \ 12.2) \quad R_{m2} = R_{mr} + (1 - p_r) \times (C_{mr} - C_{m2})^{51}$$

2. It is assumed that the safe type sticks to $m_2$. This is the case if the following incentive compatibility constraint is satisfied:

$$U_{ss}(R_{m2}, C_{m2}, a) \geq U_{ss}(R_{mr}, C_{mr}, a)$$

$$=> p_s Q_s(a) - p_s[R_{m2} + (1 - p_s)C_{m2}] \geq p_s Q_s(a) - p_s[R_{mr} + (1 - p_s)C_{mr}]$$

$$=> (A \ 12.2) \quad R_{m2} + (1 - p_s)C_{m2} \leq R_{mr} + (1 - p_s)C_{mr}$$

Insertion of (A 12.2) leads to:

$$R_{mr} + (1 - p_r) \times (C_{mr} - C_{m2}) + (1 - p_s)C_{m2} \leq R_{mr} + (1 - p_s)C_{mr}$$

$$=> C_{m2}[(1 - p_s) - (1 - p_r)] \leq C_{mr}[(1 - p_s) - (1 - p_r)]$$

$^{51}$ The condition derived thereby will be relevant in the calculation of liabilities in 2.3.3 and Annex 13.
As $C_{m2} > C_{mr}$, this must be true.

3. The MFI is indifferent between offering only $m_2$ and offering the separating pair of contracts $m_2$ and $m_r$ if its objective is equally satisfied which is

$$V_a = \lambda p_r U_{rr}(R_{ra}, C_{ra}; a) + (1 - \lambda)p_s U_{ss}(R_{sa}, C_{sa}; a),$$

i.e. the expected utilities of all borrowers, weighted by the welfare weight $\lambda \in (0,1)$ assigned to their risk type:

$$V_{a, pooled} = V_{a, separate}$$

$$\lambda p_r U_{rr}(R_{m2}, C_{m2}; a) + (1 - \lambda)p_s U_{ss}(R_{m2}, C_{m2}, a)$$

$$= \lambda p_r U_{rr}(R_{mr}, C_{mr}; a) + (1 - \lambda)p_s U_{ss}(R_{m2}, C_{m2}, a)$$

$$=> U_{rr}(R_{m2}, C_{m2}; a) = U_{rr}(R_{mr}, C_{mr}; a)$$

As this equals the condition from the first step, it is proven that the MFI is indifferent between the two options described. q.e.d.

13. Interest rates and liability after MFI entry

13.1. Proof of applicability of the zero profit condition to a separate pair of contracts

1. From the satisfaction of the incentive-compatibility constraint (see Annex 12) follows:

(A 12.2) $$R_{m2} = R_{mr} + (1 - p_r) * (C_{mr} - C_{m2})$$

In the case of joint liability $C_{mr} = 0$. Therefore:

(A 13.1.1) $$R_{mr} = R_{m2} + C_{m2}(1 - p_r)$$

2. The zero profit condition for the separating pair of contracts is:

$$\theta p_r R_{mr} + (1 - \theta)p_s [R_{m2} + C_{m2}(1 - p_s)] = \rho$$

Inserting (A 13.1.1) yields:

$$\theta p_r R_{m2} + \theta p_r C_{m2}(1 - p_r) + (1 - \theta)p_s [R_{m2} + C_{m2}(1 - p_s)] = \rho$$

$$= R_{m2}[\theta p_r + (1 - \theta)p_s] + C_{m2}[\theta p_r(1 - p_r) + (1 - \theta)p_s(1 - p_s)]$$

$$= R_{m2}\bar{p} + C_{m2}\{\bar{p} - [\theta p_r^2 + (1 - \theta)p_s^2]\}$$

This equals:

(A 13.1.2) $$\bar{p}[R_{m2} - C_{m2}(1 - p_s\beta)] = \rho$$

Which is equivalent to

(A 13.1.3) $$R_{m2} = \frac{\rho}{\bar{p}} - C_{m2}(1 - p_s\beta)$$

3. Insertion into (A 13.1.1) yields:
\[(A \ 13.1.4) \quad R_{mr} = \frac{\rho}{\beta} - C_{m2}(1 - p_s) + C_{m2}(1 - \rho_p) = \frac{\rho}{\beta} + C_{m2}(p_s - \rho_p) \]

13.2. Interest rate and joint liability

Insertion of these rates into \(V_a\) leads to

\[V_{a, \text{separate}} = \lambda p_r [p_r Q_r(a) - p_r [R_{mr} + (1 - \rho_p) C_{m2}]] + (1 - \lambda) p_s [p_s Q_s(a) - p_s [R_{m2} + (1 - \rho_s) C_{m2}]] \]

\[= \lambda p_r \left\{ \tilde{Q}(a) - p_r \left[ \frac{\rho}{\beta} + C_{m2}(p_s - \rho_p) \right] \right\} + (1 - \lambda) p_s \left\{ \tilde{Q}(a) - p_s \left[ \frac{\rho}{\beta} - C_{m2}(1 - \rho_s) + (1 - \rho_s) C_{m2} \right] \right\} \]

To maximize this, \(V_{a, \text{separate}}\) is derivated by \(\lambda\) and the derivative is set equal 0:

\[p_r \left\{ \tilde{Q}(a) - p_r \left[ \frac{\rho}{\beta} + C_{m2}(p_s - \rho_p) \right] \right\} - p_s \left\{ \tilde{Q}(a) - p_s \left[ \frac{\rho}{\beta} - C_{m2}(1 - \rho_s) + (1 - \rho_s) C_{m2} \right] \right\} = 0 \]

\[= \frac{\rho}{\beta} \left[ p_s^2 - p_r^2 \right] - \tilde{Q}(a) \left[ p_s - p_r \right] + C_{m2} \left[ -p_r^2 p_s + p_r^3 - p_s^3 + p_s^3 \beta \right] \]

\[=> C_{m2} = \frac{-\frac{\rho}{\beta} \left[ p_s^2 - p_r^2 \right] + \tilde{Q}(a) \left[ p_s - p_r \right]}{-p_r^2 p_s + p_r^3 - p_s^3(1 - \beta)} \]

\[= \frac{(p_s - p_r) \left[ \tilde{Q}(a) - \frac{\rho}{\beta} (p_s + p_r) \right]}{(p_s - p_r) \left[ p_s \beta (p_s + p_r) - (p_s^2 + p_s p_r + p_r^2) \right]} = \]

\[(A \ 13.2.1) \quad C_{m2} = \frac{\tilde{Q}(a) - \frac{\rho}{\beta} (p_s + p_r)}{p_s \beta (p_s + p_r) - (p_s^2 + p_s p_r + p_r^2)} \]

14. Numerical example with respect to the changes in the interest rates

Mookherjee and Motta set \(p_s = 0.4; p_r = 0.7; \theta = 0.3; \rho = 0.6; \pi = 0.45; \rho' = 0.68\) and and \(Q_s(a) = 1 + a^2\), and claim that the “entrance of the MFI leaves the informal lenders with safe borrowers with small landholding (approximately less than 0.4)” (2016). Calculated with the given parameters, the following table shows first that \(\delta_i\) is always lower than \(\rho' = 0.68\) which implies that not all borrowers will stay with their informal lender (case (i) in 2.3.1 does not apply). Second, \(\delta\) as presented by Mookherjee and Motta (2016) (11) will be lower than \(Q_s(a) = 1 + a^2 > 1\) regardless of \(a\). Thus, case (ii) from 2.3.1 would not apply and all borrowers would switch to the MFI. This contradicts the
proposition cited above. This claim only holds if I use the term (12) for \( \delta \), i.e. the reciprocal of (11) (see 2.3 and Annex 6).

This again implies that the term for \( \delta \) as it is presented in Mookherjee and Motta’s paper (2016) is wrong and should be replaced by (12).

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15. Derivation of liabilities for MFI contracts that pool wealth types

To derive the joint and individual liability components in MFI contracts which pool all wealth types, I follow the procedure of Annex 13 where I derived the liabilities for 2.3.3. This the MFI’s objective \( V_a \) also holds if it does not treat different landholders as separate markets.

1. As derived in Annex 11, \( V_a \) for a pooled contract and a given \( a \) is:

   \[
   V_a = \bar{\bar{a}}(a)[\lambda p_r + (1 - \lambda) p_s] - R_m[\lambda p_r^2 + (1 - \lambda) p_s^2] - C_m[\lambda p_r^2 + (1 - \lambda) p_s^2] - \lambda p_r^3 + (1 - \lambda) p_s^3]
   \]

   I assume that there are two wealth types \( a_1 \) and types \( a_2 \), whose welfare weights are \( \alpha_1 \) and \( \alpha_2 \) respectively, with \( \alpha_1 + \alpha_2 = 1 \). As \( a \) is independent of the risk type, I also assume that \( \alpha \) is independent of the risk type and its welfare weight \( \lambda \). Thus, pooling both the markets of both landholding types yields:

   \[
   V_{new} = \alpha_1 \langle \bar{\bar{a}}(a_1)[\lambda p_r + (1 - \lambda) p_s] - R_m[\lambda p_r^2 + (1 - \lambda) p_s^2] - \lambda p_r^3 + (1 - \lambda) p_s^3] \rangle + \alpha_2 \langle \bar{\bar{a}}(a_2)[\lambda p_r + (1 - \lambda) p_s] - R_m[\lambda p_r^2 + (1 - \lambda) p_s^2] - \lambda p_r^3 + (1 - \lambda) p_s^3] \rangle =
   \]
\[ \alpha_1 \bar{q}(a_1) + \alpha_2 \bar{q}(a_2) \{ \lambda p_r + (1 - \lambda) p_s \} - R_{\text{new}} \{ \lambda p_r^2 + (1 - \lambda) p_s^2 \} - C_{\text{new}} \{ \lambda p_r^2 + (1 - \lambda) p_s^2 \} \]

2. As in 2.3.3, \( R_{\text{new}} = \frac{p}{\bar{p}} - C_{\text{new}}(1 - p_s \beta) \). Insertion of this into the equation above yields:

\[ V_{\text{new}} = \{ \alpha_1 \bar{q}(a_1) + \alpha_2 \bar{q}(a_2) \{ \lambda p_r + (1 - \lambda) p_s \} - \frac{p}{\bar{p}} \{ \lambda p_r^2 + (1 - \lambda) p_s^2 \} - C_{\text{new}} p_s \beta \{ \lambda p_r^2 + (1 - \lambda) p_s^2 \} \]

3. To maximize this, \( V_{\text{new}} \) is derivated by \( \lambda \) and the derivative is set equal 0:

\[
0 = \frac{\partial}{\partial \lambda} \left[ p_s^2 - p_r^2 \right] - \{ \alpha_1 \bar{q}(a_1) + \alpha_2 \bar{q}(a_2) \} \{ p_s - p_r \}
+ C_{\text{new}} \{ -p_r^2 p_s \beta + p_r^3 - p_s^3 + p_s^3 \beta \}
\]

\[
\Rightarrow C_{\text{new}} = \frac{-\frac{p}{\bar{p}} \left( p_s^2 - p_r^2 \right) + \{ \alpha_1 \bar{q}(a_1) + \alpha_2 \bar{q}(a_2) \} \{ p_s - p_r \}}{-p_r^2 p_s \beta + p_r^3 - p_s^3 \left( 1 - \beta \right)}
= \frac{(p_s - p_r) \left\{ \left[ \alpha_1 \bar{q}(a_1) + \alpha_2 \bar{q}(a_2) \right] - \frac{p}{\bar{p}} \left( p_s + p_r \right) \right\}}{(p_s - p_r) \left\{ p_s \beta \left( p_s + p_r \right) - \left( p_s^2 + p_s p_r + p_r^2 \right) \right\} - \frac{\partial}{\partial \lambda} \left( p_s^2 + p_s p_r + p_r^2 \right)}
\]

(A 13.2.1) \( C_{\text{new}} = \frac{\left[ \alpha_1 \bar{q}(a_1) + \alpha_2 \bar{q}(a_2) \right] - \frac{p}{\bar{p}} \left( p_s + p_r \right)}{p_s \beta \left( p_s + p_r \right) - \left( p_s^2 + p_s p_r + p_r^2 \right)} \)

From \( a_1 < a_2 \) follows \( \bar{q}(a_1) < \{ \alpha_1 \bar{q}(a_1) + \alpha_2 \bar{q}(a_2) \} < \bar{q}(a_2) \), as long as project returns are increasing in \( a \).

This implies that the \( C_{\text{new}} < C_{m2,a2} \) for richer borrowers \( (a_2) \) and \( C_{\text{new}} > C_{m2,a1} \) for poorer borrowers \( (a_1) \), while \( R_{\text{new}} \) changes in the opposite direction, respectively.

Inserting \( R_{m2/new} = \frac{p}{\bar{p}} - C_{m2/new}(1 - p_s \beta) \) into \( U_i(R, C, a) = \bar{q}(a) - \left[ p_i R + p_i (1 - p_i) C \right] \) yields:

\[ U_i(C_{m2/new}, a) = \bar{q}(a) + p_i \left[ p_s C_{m2/new}(1 - \beta) - \frac{p}{\bar{p}} \right] \]

This implies that the payoff in a unified market would be higher for poorer borrowers and lower for richer borrowers.

Thus, it represents some kind of cross-subsidization.